TIME-FREQUENCY REPRESENTATIONS OF SEISMIC SIGNALS

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Introduction
A Fourier Transform of a building record contains information regarding frequency content, but it cannot resolve the exact onset of natural frequencies in an environment – all temporal resolution is contained in the phase of the signal. The spectrogram is better able to resolve temporal evolution of frequency content, but has a quality of time resolution versus frequency resolution in accordance with the uncertainty principle. To address this time/frequency resolution, several classes of Time-Frequency representations have been proposed, including wavelet methods and quadrature time-frequency distributions such as the Wigner-Ville distribution.

Time-Frequency Representations
One method of investigating a changing signal would be to take a Fourier Transform of the first half of the signal and compare the result with the second half of the signal to split the signal into four equal length frequency bins. By expanding this idea, you arrive at the spectrogram (short-time Fourier transform). Effectively, by windowing the signal and taking a Fourier transform of only a portion of the signal at a time, you can identify changes in the frequency content from one slice to the next. A window length and overlap can be selected depending on the type of signal. This assumes a certain amount of stationarity over the length of the slice (the same assumption that goes into the Fourier Transform, where the signal is assumed to be stationary and periodic). The spectrogram, however, cannot create a point where the signal is assumed to be stationary and periodic. The spectrogram, however, cannot create a point where the signal is assumed to be stationary and periodic. The spectrogram, however, cannot create a point where the signal is assumed to be stationary and periodic.

Another representation is the Wigner-Ville transform; this is similar to the Fourier Transform, but instead of decomposing signals into a basis of infinite waves, wave packets are used to represent the signal. For a discrete signal, the Wigner-Ville transform is: $X(\tau,\nu) = \sum_{t=-\infty}^{\infty} x(t) \psi(t-t') \psi^*(t-t') dt$, where $x(t)$ is the signal and $\psi(t)$ is the window function. This transform is able to identify changes in the frequency content over time.

Wigner-Ville Distribution
For a signal, $x(t)$, with analytic associate $y(t)$, the Wigner-Ville Distribution, $WVD(x,y)$, is defined as:

$$ WVD(x,y) = \int_{-\infty}^{\infty} x(\tau) y(\nu - \tau) \frac{\partial}{\partial \tau} \delta(\tau) d\tau $$

This distribution was first introduced by E. Wigner in the context of quantum mechanics [4], and later independently developed by Ville who applied the same transformation to signal processing and spectral analysis[1].

The WVD is similar to the Fourier Transform, though, instead of transforming the signal, the kernel of the WVD contains a type of auto-correlation term (in this case, the phase lag of the ambiguity function), $e^{-j\pi/2 \nu (\tau^2 + \nu \tau)}$, which results in a quadratic ambiguity function function.

In this study, time-frequency analysis techniques are applied to the analytic associates of real signals using the Cohen class of time-frequency representations. In this study, the WVD is chosen for its good properties and its ability to provide information about the signal in both the time and frequency domains. In addition, it is straightforward to extend WVD to time-varying signals.

Reduced Intereference Distribution
In general, a Reduced Interference Distribution (RID) explains to any distribution that reduces the expression of the cross-terms in the Wigner-Ville distribution. This is done by subtracting the auto-terms, which results in a reduced interference distribution.

Figure 8: Wigner-Ville Distribution. This representation shows the short-time Fourier transform of the signal along with the associated energy in the time-frequency plane. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized in the frequency representation. To illustrate this point, we apply these methods to data from instrumented structures.

Conclusions
The Wigner-Ville Distribution is optimal in many ways for creating an instantaneous estimation of frequency content. However, it has a penalty of introducing interference terms, which can be mitigated by using one of the many reduced interference distributions. With a goal of identifying the onset of changes in dynamic properties, we have developed a framework in which to apply modern time-frequency analysis techniques to data from instrumented structures.

References

Figure 1: Continuous Wavelet Transform. The wavelet transform has better time resolution than the time-frequency representation and there is only an approximate conversion between scale and frequency.

Figure 2: Continuous Wavelet Transform. The wavelet transform has better time resolution than the time-frequency representation and there is only an approximate conversion between scale and frequency.

Figure 3: Continuous Wavelet Transform. The wavelet transform has better time resolution than the time-frequency representation and there is only an approximate conversion between scale and frequency.

Figure 4: Wigner-Ville Distribution. This representation shows the short-time Fourier transform of the signal along with the associated energy in the time-frequency plane. In an ideal representation, a signal of short duration would have a narrow representation along the time axis, and the frequency content would be localized in the frequency representation.

Figure 5: Reduced Interference Distribution (Wigner-Ville). It is possible to approximate the effects of cross-term interference in a variety of ways. The RID has very crisp temporal and frequency resolution, but it is more computationally intensive than the Wigner-Ville distribution. The RID has very crisp temporal and frequency resolution, but it is more computationally intensive than the Wigner-Ville distribution.

Figure 6: Parkfield Earthquake. Millikan Library. Spectrogram at different windowing resolutions, illustrating the trade-off in temporal and frequency resolution.

Figure 7: Continuous Wavelet Transformation. Parkfield Earthquake. Millikan Library. Excellent time resolution, but not frequency resolution, due to the scale-frequency conversion being only an approximation.

Figure 9: Reduced Interference Distribution (Wigner-Ville). The RID has very crisp temporal and frequency resolution, but it is more computationally intensive than the Wigner-Ville distribution.

Application to Seismic Records
These time-frequency methods have been successfully applied to seismic records – the changes in natural frequencies can be correlated with changes in dynamic properties, such as a decrease in stiffness from earthquake damage. Obtaining a detailed, time-frequency representation of building response during earthquake excitation is one way to infer damage patterns, correlate the low-frequency transient and the baseline signal, and between the two transient signals.